Projected Written Notes from the M325K LECTURE ON TUESDAY, MARCH 19, 2024 ON SECTIONS 8.1 and 8.2: RELATIONS, INVERSE RELATIONS, and the Reflexive, Symmetric and Transitive properties, which some relations have.

CLASS #17

Test 2 is on Thursday, MARCH 21, 2024

Sections Potentially covered:

Sec 4.3

Thm (NIB) 1, 3
Using Thm (NIB) 2 in a proof

Sec 4.4, 4.5, 4.6

Proof Structure and proof methods

5.1

Σ NOTATION

Sec 5.2, 5.3, 5.4

Mathematical Induction
STRONG MATH'L Induction
W.O.P.

6.1, 6.2, 6.3

Set Theory

Elemental proofs (Disproving False Identities)

Algebraic proofs

You will not need to write a whole proof using the Well-Ordering Principle

Be able to Cite by Name:

① Theorem 4.3.1: "For all positive integers a and b, if $a \mid b$, then $a \le b$.

② Set Identities # 1, 2, 3, 4, 9, 12

Def'n: A **Relation** R from Set A to Set B

is any subset of $A \times B$,

We say "a is related to b by R" (written aRb)
$\Leftrightarrow (a,b) \in R \subseteq A \times B$

Ex 1: Let $A = \{2, 3, 5\}$, and $B = \{8, 10\}$.
Define relation R from A to B as follows:
For all $x \in A$ and for all $y \in B$,
$$x R y \Leftrightarrow x \mid y.$$

$2 R 8$ because $2 \mid 8$.

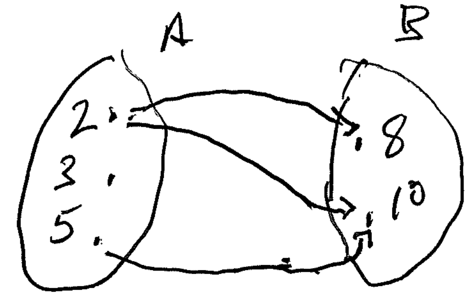
$3 \not R 8$

$5 R 10$

$2 \not R 12$ because $(2,12) \notin R$.

Relation $R = \{(2,8), (2,10), (5,10)\}$

R has this Arrow Diagram



Def'n: Given a relation R from A to B ($R \subseteq A \times B$), we define relation R^{-1} from B to A as follows:

For all $b \in B$ and for all $a \in A$,

$$b R^{-1} a \iff a R b \quad (R^{-1} \subseteq B \times A)$$

$$(b,a) \in R^{-1} \iff (a,b) \in R \subseteq (A \times B)$$

Ex: When $R = \{(2,8), (2,10), (5,10)\} \subseteq \underbrace{\{2,3,5\}}_{=A} \times \underbrace{\{8,10\}}_{=B}$

$$R^{-1} = \{(8,2), (10,2), (10,5)\} \subseteq B \times A$$

Def'n: When R is a relation from A to A, we say, "R is a relation on A" ↔ and, in this case, we call set A "The underlying set of R"

Ex 2. Let $A = \{2, 3, 4, 6, 9\}$

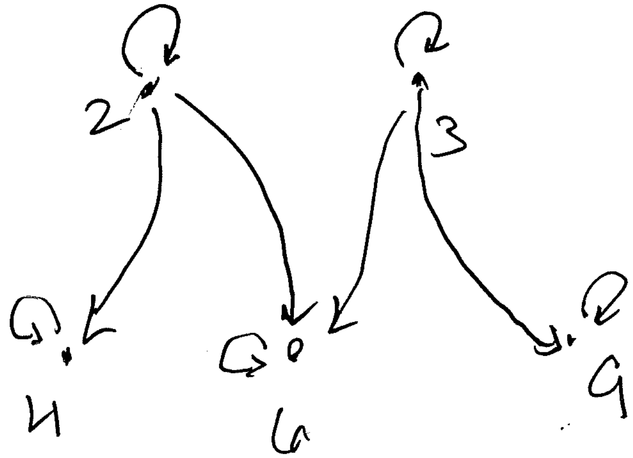
Define relation T on A as follows:

For all $a, b \in A$, $a T b \iff a \mid b$.

Some related elements:

$2 T 2$, $2 T 4$, $3 T 9$, etc.

Relation T has this Directed GRAPH:



$(2,6) \in T \subseteq A \times A$

SEC 8.2 **20.** Let $X = \{a, b, c\}$ and $\mathscr{P}(X)$ be the power set of X (the set of all subsets of X). A relation **E** is defined on $\mathscr{P}(X)$ as follows: For all A, B $\in \mathscr{P}(X)$, A **E** B $\Leftrightarrow$ the number of elements in A equals the number of elements in B.

(1-?

$\{a, b\} \in P(X)$ $\{c\} \in P(X)$

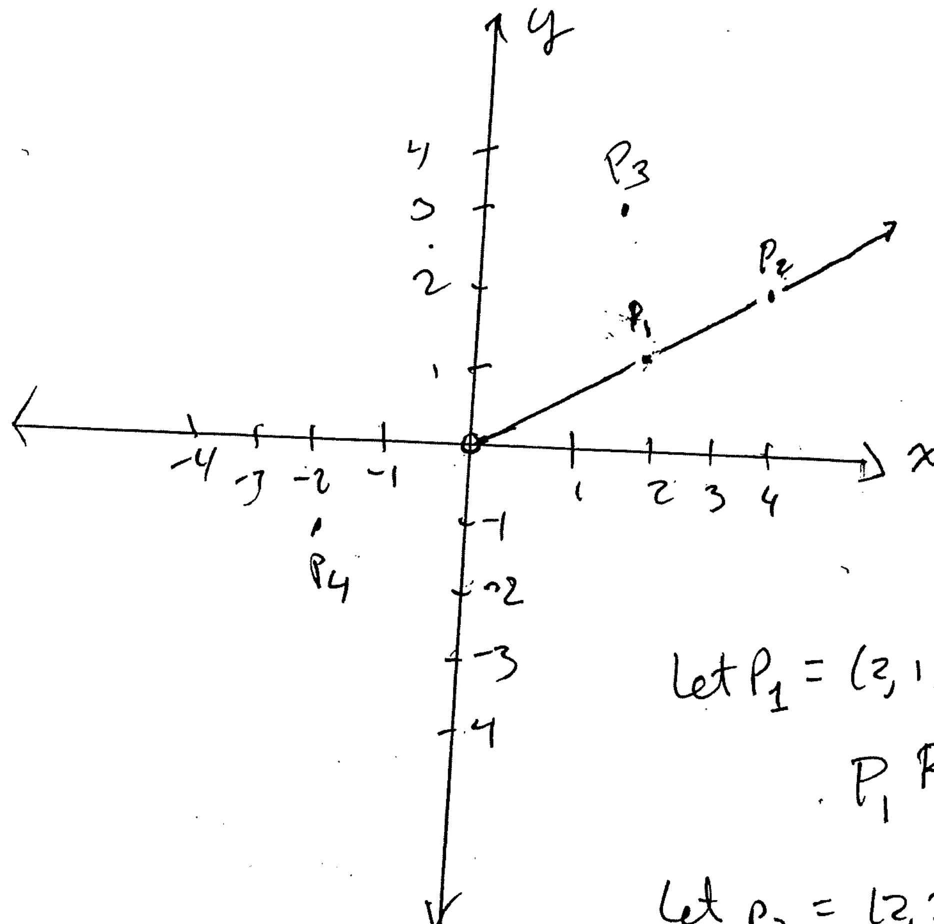
$\{a,b\} \not E \{c\}$

$\{a,b\} \; E \; \{b,c\}$

$\{b,c\} \; E \; \{b,c\}$

$\phi \, E \, \phi$

Sec. 8.2 30. Let A be the "punctured plane"; that is, A is the set of all points in the Cartesian plane except the origin $(0, 0)$. A relation R is defined on A as follows: For all p_1 and p_2 in A, $p_1 \; R \; p_2 \Leftrightarrow p_1$ and p_2 lie on the same half line emanating from the origin.



Let $P_1 = (2,1)$, $P_2 = (4,2)$

$P_1 \; R \; P_2$.

Let $P_3 = (2,3)$, $P_4 = (-2,-1)$

$P_1 \not R \; P_3$, $P_4 \not R \; P_1$

Here, Relation R is
Reflexive,
Symmetric, and
Transitive.

So, Relation R is an EQUIVALENCE RELATION.

Properties a Relation R on A might have:

R might be reflexive, R might be symmetric

R might be transitive

Def'n: Let R be a relation on set A.

(1) R is a reflexive relation

$\iff$ For all $a \in A$, aRa.

(2) R is a symmetric relation

$\iff$ For all $x, y \in A$,
if xRy, then yRx.

(3) R is a Transitive Relation

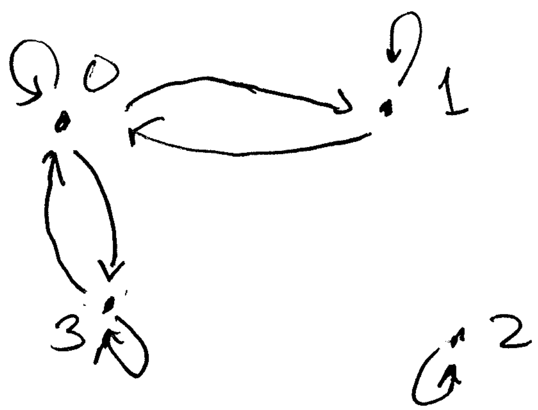
$\iff$ For all $x, y, z \in A$,
if xRy and yRz, then xRz.

R is an Equivalence Relation

$\iff$ R is Reflexive, Symmetric
AND TRANSITIVE.

Ex: Let $A = \{0, 1, 2, 3\}$

Define Relation R on A by its Directed Graph:



R is Reflexive ✓

R is Symmetric ✓

R is Not Transitive

For $a = 3, b = 0, c = 1$,

$(3R0$ and $0R1)$ and $3 \not R 1$.

$(aRb$ and $bRc)$ and $a \not R c$.

Definition of relation J on $A = \mathbb{R}$.

Define relation J on $\mathbb{R}$ as follows:

For all $x, y \in \mathbb{R}$, $x J y \Leftrightarrow xy \geq 0$.

Ex: $2 J 8$, $2 \not{J} (-3)$, $(-1) J (-1)$

$0 J 256$, since $0 \times 256 = 0 \geq 0$,

To Prove: Relation J is a symmetric Relation.

Proof: Let x and y be any real numbers.

Suppose $x J y$. [NTS: $y J x$]

$\therefore xy \geq 0$ by def'n of relation J.

$\therefore yx \geq 0$ by rules of Algebra

$\therefore y J x$ by def'n of Relation J.

$\therefore$ J is a symmetric relation by Direct Proof.

QED